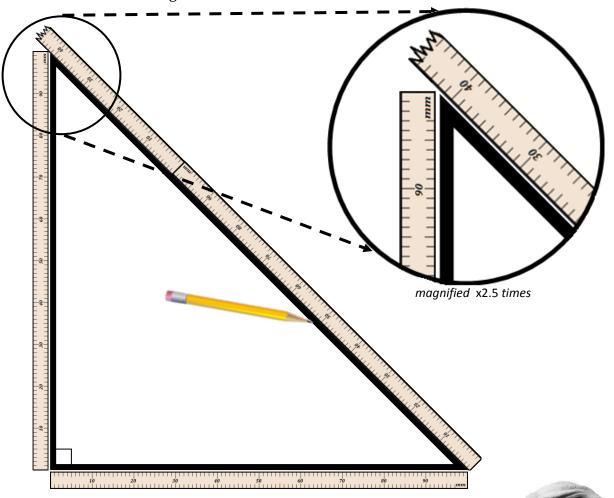


## Section 1-9 Rational & Irrational

**Rational Number**: A rational number is one that <u>can</u> represented as a ratio of  $\frac{p}{q}$ , such that p and q are both integers and  $q \neq 0$ . All rational numbers <u>can</u> be expressed as a terminating or repeating decimal. (Examples: -0.5, 0, 7,  $\frac{3}{2}$ ,  $0.2\overline{6}$ )

**Irrational Number**: An irrational number is one that  $\underline{cannot}$  represented as a ratio of  $\frac{p}{q}$ , such that p and q are both integers and  $q \neq 0$ . Irrational numbers  $\underline{cannot}$  be expressed as a terminating or repeating decimal. (Examples:  $\sqrt{3}$ ,  $\pi$ ,  $\frac{\sqrt{5}}{2}$ , e)

Irrational numbers are difficult to comprehend because they cannot be expressed easily. Consider creating a physical representation of the  $\sqrt{2}$ . Create a right triangle with each leg being exactly 1 meter. The hypotenuse should then be the length of  $\sqrt{2}$  meters.



What is interesting is, no matter how precise the ruler is, you can never measure the exact length of the hypotenuse using a metric scale. The hypotenuse will ALWAYS fall between any two lines of a metric division. This bothered early Greeks specifically the Pythagoreans. They thought it was illogical or crazy (i.e. irrational) that it was possible to draw a line of a length that could **NEVER** be measured precisely using a scale that was some integer division of the original measures. They even hid the fact that they may have known this as they believed it to be an imperfection of mathematics.

In the example above the hypotenuse is approximately 1.414213562373095048801688724209698.... meters. It can **NEVER** be written precisely as a decimal which may seem a little IRRATIONAL. The decimal description goes on forever without a repeating pattern.

A set of numbers is said to be closed under an operation if any two numbers from the original set are than combined under the operation and the solution is always in the same set as the original numbers.

**For example**, the sum of any two even numbers always results in an even number. So, the set of even numbers is closed under addition.



**For example**, the sum of any two odd numbers always results in an even number. So, the set of odd numbers is **NOT** closed under addition.



Circle One:

Circle One:

YES

Circle One:

YES

- 1. Is the set of <u>Integers</u> closed <u>under addition</u>?
- 2. Is the set of **Integers** closed **under subtraction**?
- 3. Is the set of **Integers** closed **under multiplication**?
- 4. Is the set of **Integers** closed **under division**?
- 5. Is the set of Rational Numbers closed under addition?
- 6. Is the set of Irrational Numbers closed under addition?
- 7. Is the set of **Rational Numbers** closed **under multiplication**?
- 8. Is the set of <u>Irrational Numbers</u> closed under multiplication?
- 9. Is the set of **Even Numbers** closed **under division**?
- 10. Is the set of **Odd Numbers** closed **under multiplication**?

Circic Offic.		Example of Counter Example
YES	NO	
Circle One:		Example or Counter Example
YES	NO	
Circle One:		Example or Counter Example
YES	NO	
Circle One:		Example or Counter Example
YES	NO	
Circle One:		Example or Counter Example
YES	NO	
Circle One:		Example or Counter Example
YES	NO	
Circle One:		Example or Counter Example
YES		
Circle One:		Example or Counter Example
YES	NO	

Example or Counter Example

Example or Counter Example

Example or Counter Example

NO

NO

Tell whether you think the following numbers are Rational or Irrational. 12.  $\sqrt{2+\sqrt{49}}$ 13.  $2\sqrt{27} - \sqrt{3} - \sqrt{75}$ 11.  $\sqrt{8}$ Circle One: Circle One: Circle One: Irrational Irrational Irrational Rational Rational Rational 15.  $\sqrt{12} \cdot \sqrt{3}$ 16.  $e^2$ 14.  $\pi$ Circle One: Circle One: Circle One: Irrational Rational Irrational Rational Rational Irrational 17.  $\sqrt[3]{64}$ 18.  $\sqrt[3]{24}$ 19.  $3.23131\overline{31}$ Circle One: Circle One: Circle One: Irrational Irrational Rational Irrational Rational Rational 21.  $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618034 \dots$ 22.  $81^{-\frac{3}{4}}$ 20. 3.12112111211112 ....

Circle One:

Rational

Irrational

Circle One:

Rational

Irrational

Irrational

Circle One:

Rational