Growing by Leaps and Bounds (Culminating Task)

Name_____ **Date**_____

Part I: Meet Linda

Linda's lifelong dream has been to open her own business. After working, sacrificing, and saving, she finally has enough money to open up an ice cream business. The grand opening of her business is scheduled for the Friday of Memorial Day weekend. She would like to have a soft opening for her business on the Tuesday before. The soft opening should give her a good idea of any supply or personnel issues and give her time to correct them before the big official opening.

A soft opening means that the opening of the business is not officially announced; news of its opening is just spread by word of mouth (see, not all rumors are bad!). Linda needs a good idea of when she should begin the rumor in order for it to spread reasonably well before her soft opening. She has been told that about 10% of the people who know about an event will actually attend it. Based on this assumption, if she wants to have about 50 people visit her store on the Tuesday of the soft opening, she will need 500 people to know about it.

1. Linda plans to tell one person each day and will ask that person to tell one other person each day through the day of the opening, and so on. Assume that each new person who hears about the soft opening is also asked to tell one other person each day through the day of the opening and that each one starts the process of telling their friends on the day after he or she first hears. When should Linda begin telling others about the soft opening in order to have at least 500 people know about it by the day it occurs?

2. Let x represent the day number and let y be the number of people who know about the soft opening on day x. Consider the day before Linda told anyone to be Day 0, so that Linda is the only person who knows about the opening on Day 0. Day 1 is the first day that Linda told someone else about the opening.

   a. Complete the following table.

Day	0	1	2	3	4	5
Number of people who know	1	2				

3. Graph the points from the table in part 2a.

   a. Does it make sense to connect the dots on the graph? Why or why not?

   b. What does point (5, 32) represent in this situation? Describe the point in a form of a function notation.

   c. Describe the domain of the function. What does the domain mean in this situation? What are the restrictions of the domain that arise from the context?

   d. Describe the range of the function. What does the range mean in context of the situation? What are the restrictions of the range that arise from the context?

   e. What is the y-intercept in this problem? What does the y-intercept represent in this situation?

4. The spread of a rumor or the spread of a disease can be modeled by a type of function known as exponential function; in particular, an exponential ***growth*** function. An **exponential function** has the form

   $$f(x) = a \cdot b^x,$$

   where a is a non–zero real number and b is a positive real number other than 1. An exponential growth function has a value of b that is greater than 1. Values "a" and "b" are the parameters of the function.

Write explicit and recursive equations that describe the relationship between *x* (day) and *y* (number of people who know) for the situation of spreading the news about the soft opening of Linda's ice cream store. What type of a sequence does the function represent? What do the parts of the equations represent in terms of the context?

5. Does your equation describe the relationship between day and number who know about Linda's ice cream store soft opening completely? Why or why not?

Part II: What if?

6. Predict how the graph would change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day?

7. Graph the hypothetical situation described in item 6. How did the graph change as compared to part I of the task?

8. How would the equation change if Linda had told two people each day rather than one and had asked that each person also tell two other people each day? What would be the values of *a* and *b* in this case? How does the change in parameter b affect the output values of the function?

9. How long would it take for at least 500 people to find out about the opening if the rumor spread at this new rate?

10. What if on day 0, two people instead of one knew about the business opening? What would the new equation be? What effect would this change in parameter *a* have on the graph?

Part III: The Beginning of a Business

How in the world did Linda ever save enough to buy the franchise to an ice cream store? Her mom used to say, "That Linda, why she could squeeze a quarter out of a nickel!" The truth is that Linda learned early in life that patience with money is a good thing. When she was just about 9 years old, she asked her dad if she could put her money in the bank. He took her to the bank and she opened her very first savings account.

Each year until Linda was 16, she deposited her birthday money into her savings account. Her grandparents (both sets) and her parents each gave her money for her birthday that was equal to twice her age; so on her ninth birthday, she deposited $54 ($18 from each couple).

Linda's bank paid her 3% interest, compounded quarterly. The bank calculated her interest using the following standard formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A = final amount, P = principal amount, r = interest rate, n = number of times per year the interest is compounded, and t is the number of years the money is left in the account.

1. Verify the first two rows of the following chart, and then complete the chart to calculate how much money Linda had on her 16^{th} birthday. Do not round answers until the end of the computation, then give the final amount rounded to the nearest cent.

Age	Birthday Money	Amount from previous year plus birthday money	Total at year end
8	N/A	N/A	0
9	54	0 + 54 = 54	55.63831630
10			
11			
12			
⋮	⋮	⋮	⋮

2. On her 16th birthday, the budding entrepreneur asked her parents if she could invest in the stock market. She studied the newspaper, talked to her economics teacher, researched a few companies and finally settled on the stock she wanted. She invested all of her money in the stock and promptly forgot about it. When she graduated from college on her 22nd birthday, she received a statement from her stocks and realized that her stock had appreciated an average of 10% per year. How much was her stock worth on her 22nd birthday?

3. When Linda graduated from college, she received an academic award that carried a $500 cash award. On her 22nd birthday, she used the money to purchase additional stock. She started her first job immediately after graduation and decided to save $50 each month. On her 23rd birthday she used the $600 (total of her monthly amount) savings to purchase new stock. Each year thereafter she increased her total of her savings by $100 and, on her birthday each year, used her savings to purchase additional stock. Linda continued to learn about stocks and managed her accounts carefully. On her 35th birthday she looked back and saw that her stock had appreciated at 11% during the first year after college and that the rate of appreciation increased by 0.25% each year thereafter. At age 34, she cashed in enough stock to make a down payment on a bank loan to purchase her business. What was her stock worth on her 34th birthday? Use a table like the one below to organize your calculations.

Age	Amount from previous year	Amount Linda added from savings that year	Amount invested for the year	Interest rate for the year	Amount at year end
22	998.01	500	1498.01	11.00%	1662.79
23	1662.79	600		11.25%	
24		700		11.50%	
25		800		11.75%	
⋮	⋮	⋮	⋮	⋮	⋮